Sampled-Data Control Systems
Analysis, Design and Applications

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Contents

Motivation
  Plant
  Control
  Network
  Performance optimization

Sampled-data control
  Hybrid systems
  Sampled-data control
  Markov jump linear systems

Practical applications
  Inverted pendulum
  Mass-spring-damper system

Conclusion
Motivation

- Sampled-data control in a general framework
  - Digital implementation
  - Remote control supported by internet facilities
Motivation

- From the very basic control structure ...

- \( \text{Plant } P \rightarrow \text{Modeling} \)
- \( \text{Controller } C \rightarrow \text{Control design} \)
Motivation

... to networked control!

- **Plant** $\mathcal{P}$ $\rightarrow$ Modeling
- **Controller** $C$ $\rightarrow$ Control design
- **Network** $\rightarrow$ Signal quality and limitations
The plant $\mathcal{P}$ is linear time invariant - LTI

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) \\
z(t) &= Cx(t) + Du(t)
\end{align*}
\]

- $x_0 \in \mathbb{R}^n$ is the initial condition
- $x(t) : \mathbb{R}_+ \rightarrow \mathbb{R}^n$ is the state
- $u(t) : \mathbb{R}_+ \rightarrow \mathbb{R}^m$ is the control
- $z(t) : \mathbb{R}_+ \rightarrow \mathbb{R}^q$ is the controlled output
- $w(t) : \mathbb{R}_+ \rightarrow \mathbb{R}^p$ is the exogenous perturbation input
Control

- Communication channel modeling! – Operator $\mathcal{R}$

$$f(t) \rightarrow \mathcal{R}f(t)$$

trasmitted signal \hspace{1cm} received signal

- Effective remote control applied to the plant

$$u(t) = \mathcal{R}C(\mathcal{R}x)(t)$$

- The state $x(t)$ is measured and the controller receives $\mathcal{R}x(t)$
- The transmitted controller output $C(\mathcal{R}x(t))$ provides $u(t)$
Network

- **Bandwidth limitation:** Data transmission with maximum frequency $1/T$. Successive sampling instants satisfy $T_k = t_{k+1} - t_k \geq T, \forall k \in \mathbb{N}$, (Matveev and Savkin, 2009)

\[ Rf(t) = f(t_k), \forall t \in [t_k, t_{k+1}), \forall k \in \mathbb{N} \]

- Sampled-data control operator, (Chen and Francis, 1995) and (Ichicawa and Katayama, 2001)
Packet dropout: Network information loss imposes

\[ \mathcal{R}f(t) = \Gamma_{\theta(t)} f(t) \]

where \( \theta(t) \in K = \{1, \ldots, N\} \) are the states of a Markov chain, (Costa, Fragoso and Todorov, 2013)

- Control synthesis is merged in a stochastic framework!
Performance optimization

- $\mathcal{H}_2$ performance: From zero initial condition

$$J_2(u) = \sum_{l=1}^{p} \| z_l \|_2^2 \quad \text{subject to} \quad w(t) = e(t) \delta(t)$$

- $\mathcal{H}_\infty$ performance: From zero initial condition

$$J_\infty(u) = \inf_{\gamma} \left\{ \gamma^2 : \| z \|_2^2 \leq \gamma^2 \| w \|_2^2 \right\}, \forall w \in \mathcal{L}_2$$

worst case perturbation. It passes through the network?
Performance optimization

- The main goal is to solve

\[ \inf_{u \in \mathcal{U}} J_\alpha(u) \]

where:

- \( \alpha \in \{2, \infty\} \)
- \( u \in \mathcal{U} \) define feasible signals subject to the network limitations
- **Solution strategy** → eliminate the constraint \( u \in \mathcal{U} \) and redefine an equivalent design problem
Sampled-data control

- Consider uniform sampling $T = t_{k+1} - t_k > 0$, $k \in \mathbb{N}$ and

  $$u \in \mathcal{U} \iff u(t) = u(t_k), \quad \forall t \in [t_k, t_{k+1})$$

- Is it possible to determine a discrete time equivalent system?

  $$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0$$

  $$z(t) = Cx(t) + Du(t)$$

  $\Downarrow$

  $$\int_0^\infty z(t)'z(t)dt = \sum_{k=0}^\infty z(t_k)'z(t_k)$$

  $\Uparrow$

  $$x(t_{k+1}) = A_d x(t_k) + B_d u(t_k), \quad x(0) = x_0$$

  $$z(t_k) = C_d x(t_k) + D_d u(t_k)$$
Sampled-data control

The following fundamental formula

\[
\begin{bmatrix}
    x(t) \\
    u(t)
\end{bmatrix} = e^{F(t-t_k)} \begin{bmatrix}
    x(t_k) \\
    u(t_k)
\end{bmatrix}, \quad \forall t \in [t_k, t_{k+1})
\]

provides the affirmative answer

\[
F = \begin{bmatrix}
    A & B \\
    0 & 0
\end{bmatrix} \quad \rightarrow \quad e^{Ft} = \begin{bmatrix}
    A_d & B_d \\
    0 & I
\end{bmatrix}
\]

\[
G = \begin{bmatrix}
    C \\
    D
\end{bmatrix} \quad \rightarrow \quad \int_0^T e^{F't} G' Ge^{Ft} dt = \begin{bmatrix}
    C'_d \\
    D'_d
\end{bmatrix} \begin{bmatrix}
    C'_d \\
    D'_d
\end{bmatrix}'
\]
Sampled-data control

- Optimal solution of the $\mathcal{H}_2$ control problem

$$\inf_{u \in \mathcal{U}} J_2(u) = \min_L \left\| (C_d + D_d L) (zI - (A_d + B_d L))^{-1} E \right\|_2^2$$

- The optimal matrix gain follows from the stabilizing solution of a discrete time algebraic Riccati equation
- The optimal control is of the form

$$u(t) = Lx(t_k), \quad \forall t \in [t_k, t_{k+1})$$
Sampled-data control

- Consider the plant with exogenous input $w \in \mathcal{L}_2$

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) , \quad x(0) = 0 \\
z(t) &= Cx(t) + Du(t)
\end{align*}
\]

The fundamental formula now becomes

\[
\begin{bmatrix}
\dot{x}(t) - \int_{t_k}^{t} e^{A(t-\tau)} Ew(\tau) d\tau \\
u(t)
\end{bmatrix} = e^{F(t-t_k)}
\begin{bmatrix}
x(t_k) \\
u(t_k)
\end{bmatrix}
\]

valid for all $t \in [t_k, t_{k+1})$

\[\downarrow\]

Impossible to determine a discrete-time equivalent system unless $w(t) = w(t_k), \quad \forall t \in [t_k, t_{k+1})$
Sampled-data control

- For this class of not too severe external perturbations

\[ w \in L_{2T} \subset L_2 \]

a new performance index is introduced

\[ J_T(u) = \inf_{\gamma} \{ \gamma^2 : \|z\|_2^2 \leq \gamma^2 \|w\|_2^2 \}, \quad \forall w \in L_{2T} \]

which satisfies

\[ \inf_{u \in U} J_\infty(u) \geq \inf_{u \in U} J_T(u) \]

\[ \downarrow \]

Is this lower bound useful in the context of \( H_\infty \) control?
Sampled-data control

- (Sub) Optimal solution of the $\mathcal{H}_\infty$ control problem

$$\inf_{u \in U} \mathcal{J}_T(u) = \min_L \left\| (C_d + D_d L) \left( zI - (A_d + B_d L) \right)^{-1} E_d \right\|_\infty^2$$

- Matrix $E_d$ is obtained as before by replacing $B \rightarrow [B \ E]$
- The optimal matrix gain follows from the stabilizing solution of a discrete time algebraic Riccati equation
- The optimal control is of the form

$$u(t) = Lx(t_k), \ \forall t \in [t_k, t_{k+1})$$
Sampled-data control

- Sampled-data optimal control in a general framework

Hybrid Systems
Sampled-data control

- Sampled-data optimal control in a general framework

\[ \downarrow \]

Hybrid Systems

+ Bellman’s Principle of Optimality

\[
\text{cost to go}_{t_k} = \inf_{u(t_k)} \left\{ \text{cost to go}_{t_{k+1}} + \text{cost}_{t_k}^{t_{k+1}} \right\}
\]
Hybrid systems

- Adopting uniform sampling $T = t_{k+1} - t_k > 0$, $k \in \mathbb{N}$ and

\[
 u \in \mathcal{U} \leftrightarrow u(t) = Lx(t_k), \ \forall t \in [t_k, t_{k+1})
\]

the closed-loop sampled-data system can be rewritten as

\[
\begin{align*}
\dot{\xi}(t) &= \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \xi(t) + \begin{bmatrix} E \\ 0 \end{bmatrix} w(t) \\
z(t) &= \begin{bmatrix} C & D \end{bmatrix} \xi(t) \\
\xi(t) &= \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \leftrightarrow \xi(t_k) = \begin{bmatrix} I & 0 \\ L & 0 \end{bmatrix} \xi(t_k^-), \ \forall t \in [t_k, t_{k+1})
\end{align*}
\]
Hybrid systems

Let us consider a generic hybrid system of the form

\[
\dot{\xi}(t) = F\xi(t) + Jw(t)
\]

\[
z(t) = G\xi(t)
\]

\[
\xi(t_k) = H\xi(t_k^-)
\]

**Necessary and sufficient condition** for asymptotic stability follows from the discrete-time process

\[
\xi(t_k) \to \xi(t_k^-) \to \xi(t_{k+1}), \ \forall k \in \mathbb{N}
\]

**Performance indexes** \( J_2(u) \) and \( J_\infty(u) \) calculation
\( H_2 \) performance

- If the matrix differential equation

\[
\dot{P}(t) + F'P(t) + P(t)F = -G'G
\]

admits a solution in the time interval \( t \in [0, T] \) satisfying the boundary conditions

\[
P(0) < S^{-1}, \quad P(T) > H'S^{-1}H
\]

for some symmetric matrix \( S > 0 \), then the hybrid system is asymptotically stable and satisfies

\[
\mathcal{J}_2(u) < \text{Tr}(J'H'S^{-1}HJ)
\]
\( \mathcal{H}_2 \) performance

- Matrix \( S > 0 \) satisfies the Lyapunov inequality:

\[
e^{F' T} H' S^{-1} H e^{F T} < S^{-1} - \int_0^T e^{F' t} G' G e^{F t} dt \geq 0
\]

which admits a solution if and only if \( He^{FT} \) is Schur stable.

- The optimal sampled-data \( \mathcal{H}_2 \) control follows from

\[
\inf_{L, S > 0} \text{Tr}(J' H' S^{-1} H J) \rightarrow \text{CONVEX}?
\]

Notice that \( H \) depends on the state feedback gain \( L \).
\( \mathcal{H}_\infty \) performance

- If the matrix differential equation

\[
\dot{P}(t) + F'P(t) + P(t)F + \gamma^{-2}P(t)JJ'P(t) = -G'G
\]

admits a solution in the time interval \( t \in [0, T] \) satisfying the boundary conditions

\[ P(0) < S^{-1}, \quad P(T) > H'S^{-1}H \]

for some symmetric matrix \( S > 0 \), then the hybrid system is asymptotically stable and satisfies

\[ \mathcal{J}_\infty(u) < \gamma^2 \]
We have to solve a nonlinear differential equation!

There exists a solution $P(t), \forall t \in [0, T]$ if and only if the algebraic Riccati equation

$$FQ + QF' + \gamma^{-2} JJ' + QG' GQ = 0$$

admits a solution. Defining $\bar{F} = F + QG' G$

- In general $Q$ is sign indefinite
- The following matrices can be readily calculated

$$\bar{F}_d = e^{\bar{F} T}, \quad \int_0^T e^{\bar{F}' \tau} G' Ge^{\bar{F}\tau} d\tau = \bar{G}'_d \bar{G}_d$$
$\mathcal{H}_\infty$ performance

- Solution of the Riccati differential equation
- The $\mathcal{H}_\infty$ two boundary value problem admits a solution if and only if there exist $W$, $S$ such that $S > Q$ and

$$\begin{bmatrix} W & WH' \\ HW & S \end{bmatrix} > 0$$

$$\begin{bmatrix} W - Q & 0 \\ 0 & I \end{bmatrix} > \begin{bmatrix} \bar{F}_d \\ \bar{G}_d \end{bmatrix} (S - Q) \begin{bmatrix} \bar{F}_d \\ \bar{G}_d \end{bmatrix}'$$
$\mathcal{H}_\infty$ performance

- Jointly convex problem in the matrix variables $(W, S)$
- The limit case
  \[ \gamma \to +\infty , \ Q = 0 \]
  provides a solution to the $\mathcal{H}_2$ two boundary value problem
- Including the matrix gain $L$, in the set of variables, the problem remains CONVEX?
\( \mathcal{H}_\infty \) performance

- **Important:** Continuous-time systems characterized by \( H = I \)
- From the solution of the \( \mathcal{H}_\infty \) two boundary value problem

\[
P(t) = P = S^{-1}, \ \forall t \in [0, T]
\]

where \( P > 0 \) is the stabilizing solution of the algebraic Riccati equation

\[
F'P + PF + \gamma^{-2}PJJ'P = -G'G
\]

the classical results are recovered

\[
\mathcal{J}_2 = \| G(sI - F)^{-1} J \|_2^2
\]

\[
\mathcal{J}_\infty = \| G(sI - F)^{-1} J \|_\infty^2
\]
Sampled-data control

- Main property of the Riccati equation solution $Q$

$$
\bar{F} = \begin{bmatrix} F \\ \bar{Q} \end{bmatrix} + \begin{bmatrix} Q \\ 0 \end{bmatrix} \begin{bmatrix} G' \\ 0 \end{bmatrix}
$$

matrices $F$ and $\bar{F}$ have the same structure!

$$
\bar{F}_d = \begin{bmatrix} \bar{A}_d & \bar{B}_d \\ 0 & I \end{bmatrix}, \quad \bar{G}_d = \begin{bmatrix} \bar{C}_d & \bar{D}_d \end{bmatrix}
$$
Sampled-data control

- The determination of the state feedback gain $L$ requires the block structure

$$S = \begin{bmatrix} X & Y \\ Y' & Z \end{bmatrix}$$

- The $\mathcal{H}_\infty$ two boundary value problem admits a solution if and only if there exists $S > 0$ such that $S > Q$ and

$$\begin{bmatrix} X - \bar{Q} & 0 \\ 0 & I \end{bmatrix} > \begin{bmatrix} \bar{A}_d & \bar{B}_d \\ \bar{C}_d & \bar{D}_d \end{bmatrix} \begin{bmatrix} X - \bar{Q} & Y \\ Y' & Z \end{bmatrix} \begin{bmatrix} \bar{A}_d & \bar{B}_d \\ \bar{C}_d & \bar{D}_d \end{bmatrix}' \quad (\ast)$$

In the affirmative case $L = Y'X^{-1}$. 

30 / 50
Sampled-data control

- $\mathcal{H}_\infty$ optimal control

$$\inf_{S>0, S>Q, \gamma} \{ \gamma^2 : (\ast) \}$$

- $\mathcal{H}_2$ optimal control

$$\inf_{S>0} \{ \text{Tr}(E'X^{-1}E) : (\ast) \}$$

adopting the limit case solution $\gamma \to +\infty$, $Q = 0$

Both are convex problems expressed by LMIs
Example

A second order system with state space realization

\[
A = \begin{bmatrix} 0 & 1 \\ -6 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

\[
C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

has been treated. Three problems have been solved:

- $\mathcal{H}_\infty$ optimal control
- $\mathcal{H}_\infty$ optimal control with $w \in \mathcal{L}_{2T} \subset \mathcal{L}_2$
- $\mathcal{H}_\infty$ suboptimal control
Example

\[
\log_{10}(\gamma)
\]
Nonuniform sampling

- Time distance between successive samplings is uncertain
  \[ t_{k+1} - t_k = T_k \in \mathcal{T}, \forall k \in \mathbb{N} \]

- $\mathcal{H}_2$ optimal control
  \[
  \inf_{S > 0} \left\{ \operatorname{Tr}(E'X^{-1}E) : \begin{array}{c}
  (*) \\
  \psi(S,T) > 0
  \end{array} \right\}, \forall T \in \mathcal{T}
  \]

LMI $\implies$ convex set whenever $T \in \mathcal{T}$ is fixed!
Nonuniform sampling

- Globally convergent algorithm!

\[ J_{\ell+1} \geq J_{\ell}, \quad \ell \in \{0, 1, \cdots\} \]

- Example

\[
\begin{align*}
A &= \begin{bmatrix} 0 & 1 \\ -16 & 4.8 \end{bmatrix}, & B &= \begin{bmatrix} 0 \\ 16 \end{bmatrix}, & E &= \begin{bmatrix} 0 \\ 16 \end{bmatrix} \\
C &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, & D &= \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, & T &= (0, \pi/25)
\end{align*}
\]

- \( L = \begin{bmatrix} 0.8430 & -0.4781 \end{bmatrix} \rightarrow \gamma = 24.407, \quad (\text{Suplin et al., 2007}) \)
- \( L = \begin{bmatrix} -2.0620 & -0.8793 \end{bmatrix} \rightarrow \gamma = 1.0401 \)
Markov jump linear systems

- Consider a MJLS with state space realization

\[
\begin{align*}
\dot{x}(t) &= A_{\theta(t)}x(t) + B_{\theta(t)}u(t) + E_{\theta(t)}w(t) \\
z(t) &= C_{\theta(t)}x(t) + D_{\theta(t)}u(t)
\end{align*}
\]

- \( \theta(t) \in \mathbb{K} \) is a Markovian process defined by

\[
P(\theta(t + h) = j | \theta(t) = i) = \delta_{i-j} + \lambda_{ij}h + \mathcal{O}(h)
\]

- \( x(0) = 0 \)
- \( \theta(0) = \theta_0 \) with probability \( P(\theta_0 = i) = \pi_{i0}, \forall i \in \mathbb{K} \)
- Performance indexes are defined accordingly to

\[
\|\eta\|_2^2 = \int_0^\infty \mathbb{E}\{\eta(t)'^t\eta(t)\}dt
\]
Hybrid Markov jump linear systems

- Hybrid Markov jump linear system

\[
\begin{align*}
\dot{\xi}(t) &= F_{\theta(t)}\xi(t) + J_{\theta(t)}w(t) \\
z(t) &= G_{\theta(t)}\xi(t) \\
\xi(t_k) &= H_{\theta(t_k)}\xi(t_{k^-})
\end{align*}
\]

- Sampling instants \( T = t_{k+1} - t_k > 0, \forall k \in \mathbb{N} \) are independent of the Markov process
- Jumps may occur inside the sampling interval
- State feedback control design of the form

\[
u(t) = u(t_k) = L_{\theta(t_k)}x(t_k), \forall t \in [t_k, t_{k+1})
\]

- Impossible to determine an equivalent discrete-time system!
$\mathcal{H}_2$ performance

- If the coupled matrix differential equations

$$\dot{P}_i(t) + F_i' P_i(t) + P_i(t) F_i + \sum_{j \in K} \lambda_{ij} P_j(t) = -G_i' G_i$$

for $i \in K$ admit a solution in the time interval $t \in [0, T]$ satisfying the boundary conditions

$$P_i(0) < S_i^{-1}, \quad P_i(T) > H_i' S_i^{-1} H_i, \quad i \in K$$

for some symmetric matrices $S_i > 0$, then the hybrid Markov jump linear system is mean square stable and satisfies

$$J_2(u) < \sum_{i \in K} \pi_{i0} \text{Tr}(J_i' H_i' S_i^{-1} H_i J_i)$$
The previous result characterizes the optimal solution

As before, the following matrices are defined

$$\tilde{F}_i = F_i + \left(\lambda_{ii}/2\right)I \implies \tilde{F}_{di} = e^{\tilde{F}_iT}, \; i \in K$$

$$\bar{G}'_{di} \bar{G}_{di} = \int_0^T e^{\tilde{F}_i \tau} \begin{pmatrix} G'_i G_i + \sum_{j \neq i \in K} \lambda_{ij} P_j(\tau) \\ \geq 0 \end{pmatrix} e^{\tilde{F}_i \tau} d\tau, \; i \in K$$
\( \mathcal{H}_2 \) performance

- The determination of the state feedback gains \( L_i, \ i \in \mathbb{K} \) requires the block structure

\[
S_i = \begin{bmatrix} X_i & Y_i \\ Y_i' & Z_i \end{bmatrix} > 0, \ i \in \mathbb{K}
\]

and to solve

\[
\inf_{S_i > 0} \sum_{i \in \mathbb{K}} \pi_{i0} \text{Tr}(E_i'X_i^{-1}E_i)
\]

\[
\begin{bmatrix} X_i & 0 \\ 0 & I \end{bmatrix} > \begin{bmatrix} \bar{A}_{di} & \bar{B}_{di} \\ \bar{C}_{di} & \bar{D}_{di} \end{bmatrix} S_i \begin{bmatrix} \bar{A}_{di} & \bar{B}_{di} \\ \bar{C}_{di} & \bar{D}_{di} \end{bmatrix}', \ i \in \mathbb{K}
\]

\( \Downarrow \)

In the affirmative case \( L_i = Y_i'X_i^{-1}, \ i \in \mathbb{K} \)
$H_2$ performance

- **Important facts:**
  - $N$ uncoupled subproblems
  - Matrices $\bar{G}_{di}, i \in K$ are coupled through the dependence of $P_j, j \neq i \in K$. For this reason a **globally convergent algorithm** has been developed

\[
S_{i(\ell+1)} \geq S_{i(\ell)} \geq 0, \ell \in \{0, 1, \cdots\}
\]

- There is no difficulty to treat $H_\infty$ control design problems
Inverted pendulum

- Inverted pendulum without friction

whose vertical displacement \( \theta_v = \phi - \pi/2 \) follows from the linearized model.

\[
(M + m)\ddot{x}_h - m\ell\ddot{\theta}_v = u
\]
\[
\ell\ddot{\theta}_v - \ddot{x}_h - g\theta_v = 0
\]
Inverted pendulum

- **Sampled-data $\mathcal{H}_2$ optimal** - uniform sampling

  \[
  T = 500 \text{ [ms]}
  \]

  \[
  \Downarrow
  \]

  \[
  L = [1.3023, 3.3221, -159.1264, -46.8910] \implies J_2(u) = 156.6714
  \]

- **Sampled-data $\mathcal{H}_2$ guaranteed** - nonuniform sampling

  \[
  T_k \in \mathcal{T} = [300, 700] \text{ ms} \quad \forall k \in \mathbb{N}
  \]

  \[
  \Downarrow
  \]

  \[
  L = [0.6539, 1.9751, -146.6085, -43.0491] \implies J_2(u) = 370.4710
  \]
Inverted pendulum

- Time simulation - $T_k$ uniformly distributed inside $\mathcal{T}$
Two masses without friction connected with a damper and two springs, (Lutz, 2014)

The control signal is transmitted through a network:

- **Bandwidth limitation**: $1/T = 2$ [Hz]
- **Packet dropout**: Two Markov modes $\mathbb{K} = \{1, 2\}$ corresponding to package loss and transmission success defined by a transition rate matrix $\Lambda$ such that

$$e^{\Lambda T_0} = \begin{bmatrix} 0.85 & 0.15 \\ 0.10 & 0.90 \end{bmatrix}, \quad T_0 = 20$[ms]

- Initial probability $\pi_0 = [0.4 \ 0.6]'$
Mass-spring-damper system

- **Sampled-data $\mathcal{H}_\infty$ optimal control - uniform sampling**

$$T = 500 \text{ [ms]}$$

$$\downarrow$$

$$J_\infty(u) = \gamma_{opt}^2, \quad \gamma_{opt} = 1.5351$$

Considering $\gamma = 1.6$ we have calculated state feedback gains

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 0.8653 & 1.0451 & -0.6580 & -3.1297 \\ 0.7204 & 0.7531 & -0.5783 & -2.7536 \end{bmatrix}$$
Mass-spring-damper system

- Algorithm evolution for $\gamma = 1.6$
Mass-spring-damper system

Monte Carlo simulation of 500 runs provides the trajectories of the closed-loop system excited by the exogenous perturbation \( w(t) = \sin(6.38t) \), for \( t \in [0, 2] \) [s] and \( w(t) = 0 \) elsewhere, (Leon-Garcia, 2008)
Conclusion

- **Linear filtering:** Optimal filtering in the context of networked systems

- **Dynamic output feedback control:** Optimal control under partial information taking into account bandwidth limitation and packet dropout occurrence

- **Switched control:** Optimal decision rule for the determination of $T_k \in \mathcal{T}$ at each $k \in \mathbb{N}$ towards performance optimization.
Thank you very much for your attention